# Exponential Localization of Wannier Functions in Insulators

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The question of the exponential localization of generalized Wannier functions has long been one of the last open problems of one-body solid-state physics. The exponential localization of Wannier functions in two and three dimensions is important from a theoretical point of view, because it implies the existence of multipoles of all orders, and from a computational point of view, because it ensures the exponential convergence of some numerical algorithms.
We proved recently [1] that 2D and 3D generalized Wannier functions decay exponentially in insulators if and only if the single Chern number in 2D and the three Chern numbers in 3D are zero.
After a brief history of the problem, we sketch the steps of the proof, that boils down to a recent theorem on the topology of the Brillouin zone [2]. The role played by the Berry connection and the Chern numbers in the proof of the theorem is explained. Some of the practical consequences of exponential decay are presented.

[1] Ch. Brouder, G. Panati, M. Calandra, Ch. Mourougane, N. Marzari, Phys. Rev. Lett. **98**, 046402 (2007).

[2] G. Panati, Ann. Henri Poincaré **8**, (2007).